Academic Standards for Mathematics

January 29, 2010 -- FINAL DRAFT
Secondary Standards
Grades 8, 11
Algebra I, Geometry, Algebra II

Pennsylvania Department of Education

These standards are offered as a voluntary resource for Pennsylvania's schools and await action by the State Board of Education.
The course level standards are offered as a voluntary resource for Pennsylvania's schools.

SECONDARY STANDARDS
GRADES 8, 11
Algebra I, Geometry, Algebra II

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SECONDARY STANDARDS
GRADES 8, 11
Algebra I, Geometry, Algebra II

V. INTRODUCTION

This document includes Mathematics Standards:

- ◊ 2.1. Numbers, Number Systems and Number Relationships
- ◊ 2.2. Computation and Estimation
- ◊ 2.3. Measurement and Estimation
- ◊ 2.4. Mathematical Reasoning and Connections
- ◊ 2.5. Mathematical Problem Solving and Communication
- ◊ 2.6. Statistics and Data Analysis
- ◊ 2.7. Probability and Predictions
- ◊ 2.8. Algebra and Functions
- ◊ 2.9. Geometry
- ◊ 2.10. Trigonometry
- ◊ 2.11. Concepts of Calculus

The Mathematics Standards describe what students should know and be able to do in Algebra I, Algebra II, and Geometry courses. They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school.

With each Standard divided into conceptual strands, this document avoids repetition of learned skills and makes an obvious progression across grade levels less explicit. Teachers shall expect that students know and can apply the concepts and skills expressed at the preceding level. Consequently, previous learning is reinforced but not re-taught.

Students who achieve these mathematical standards will be able to communicate mathematically. Although it is an interesting and enjoyable study for its own sake, mathematics is most appropriately used as a tool to help organize and understand information from other academic disciplines. Because our capacity to deal with all things mathematical is changing rapidly, students must be able to bring the most modern and effective technology to bear on their learning of mathematical concepts and skills.

A glossary is included to assist the reader in understanding terminology contained in the standards. Words in bold faced text are included in the glossary

2.1. Numbers, Number Systems and Number Relationships					
	2.1.8. GRADE 8	**2.1.A1. ALGEBRA I**	**2.1.G. GEOMETRY**	**2.1.A2. ALGEBRA II**	**2.1.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Count and Compare Numbers	2.1.8.A. Model and compare values of **integers** and **rational numbers**.	2.1.A1.A. Model and compare values of **irrational numbers**.	2.1.G.A. Model and compare values of **irrational numbers**.	2.1.A2.A. Model and compare values of **complex** numbers.	2.1.11.A. Model and compare values of **irrational** and **complex numbers.**
Represent Numbers in Equivalent Forms	2.1.8.B. Represent and use numbers in **equivalent** forms (e.g., **integers**, fractions, decimals, percents, **exponents**, **scientific notation**, square **roots**, **absolute values**).	2.1.A1.B. Use factoring to create **equivalent forms** of **polynomial**s.	2.1.G.B. Intentionally Blank	2.1.A2.B. Use factoring to create **equivalent forms** of **polynomial**s.	2.1.11.B. Use factoring to create **equivalent** forms of **polynomials**.
Concepts of Numbers and Relationships	2.1.8.C. Use **ratio** and **proportion** to model relationships between quantities.	2.1.A1.C. Use **ratio** and **proportion** to model relationships between quantities.	2.1.G.C. Use **ratio** and **proportion** to model relationships between quantities.	2.1.A2.C. Intentionally Blank	2.1.11.C. Intentionally Blank

2.1. Numbers, Number Systems and Number Relationships					
	2.1.8. GRADE 8	**2.1.A1. ALGEBRA I**	**2.1.G. GEOMETRY**	**2.1.A2. ALGEBRA II**	**2.1.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Place Value	2.1.8.D. Extend place-value concepts to represent large numbers using **exponential, scientific,** and calculator notation.	2.1.A1.D. Use exponential, scientific, and calculator notation to represent any **rational number**.	2.1.G.D. Intentionally Blank	2.1.A2.D. Use **exponential notation** to represent any **rational number**.	2.1.11.D. Use **exponential, scientific,** and calculator notation to represent any **rational number**.
Number Theory	2.1.8.E. Apply concepts of **prime** and **composite** numbers to calculate GCFs (Greatest Common Factor) and LCMs (Least Common Multiple) of numbers.	2.1.A1.E. Apply the concepts of **prime** and **composite monomials** to determine GCFs (Greatest Common Factor) and LCMs (Least Common Multiple) of **monomials**.	2.1.G.E. Intentionally Blank	2.1.A2.E. Intentionally Blank	2.1.11.E. Apply the concepts of **prime** and **composite polynomials** to determine GCFs (Greatest Common Factor) and LCMs (Least Common Multiple) of **polynomials**.
Concepts and Applications of Operations	2.1.8.F. Understand the concepts of **exponents** and **roots** and use the **inverse** relationships between **exponents** and **roots** to determine unknown quantities in **equations**.	2.1.A1.F. Extend the concept and use of **inverse operations** to determine unknown quantities in linear and **polynomial equations**.	2.1.G.F. Intentionally Blank	2.1.A2.F. Understand the concepts of **exponential** and **logarithmic** forms and use the **inverse** relationships between **exponential** and **logarithmic expression** to determine unknown quantities in **equations**.	2.1.11.F. Understand the concepts of **exponential** and **logarithmic** forms and use the **inverse** relationships between **exponential** and **logarithmic expression** to determine unknown quantities in **equations**.

2.2. Computation and Estimation					
	2.2.8. GRADE 8	**2.2.A1. ALGEBRA I**	**2.2.G. GEOMETRY**	**2.2.A2. ALGEBRA II**	**2.2.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Fluency in Basic Facts	2.2.8.A. Intentionally Blank	2.2.A1.A. Intentionally Blank	2.2.G.A. Intentionally Blank	2.2.A2.A. Intentionally Blank	2.2.11.A. Intentionally Blank
Computation	2.2.8.B. Add, subtract, multiply, and divide different kinds and forms of **rational numbers** including **integers**, decimal fractions, percents**,** and proper and improper fractions.	2.2.A1.B. Intentionally Blank	2.2.G.B. Intentionally Blank	2.2.A2.B. Intentionally Blank	2.2.11.B. Intentionally Blank
Evaluate Numerical Expressions	2.2.8.C. Use the **order of operations** to evaluate numerical **expressions**.	2.2.A1.C. Evaluate numerical **expressions** that include the four basic operations and operations of **powers** and **roots**, **reciprocals**, opposites, and **absolute values**.	2.2.G.C. Intentionally Blank	2.2.A2.C. Evaluate numerical **expressions** of complex numbers that include the four basic operations and operations of **powers**, opposites, conjugates, and **absolute values**.	2.2.11.C. Evaluate numerical **expressions** that include the four basic operations and operations of **powers** and **roots**, **reciprocals**, opposites, and **absolute values**.

2.2. Computation and Estimation					
	2.2.8. GRADE 8	**2.2.A1. ALGEBRA I**	**2.2.G. GEOMETRY**	**2.2.A2. ALGEBRA II**	**2.2.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Numerical Estimation	2.2.8.D. Estimate the values of **irrational numbers** and the results from calculations with basic operations of fractions and percents and check the **reasonableness** of those estimates.	2.2.A1.D. Intentionally Blank	2.2.G.D. Intentionally Blank	2.2.A2.D. Intentionally Blank	2.2.11.D. Intentionally Blank

2.3. Measurement and Estimation					
	2.3.8. GRADE 8	**2.3.A1. ALGEBRA I**	**2.3.G. GEOMETRY**	**2.3.A2. ALGEBRA II**	**2.3.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Concept of Measurement	2.3.8.A. Intentionally Blank	2.3.A1.A. Intentionally Blank	2.3.G.A. Intentionally Blank	2.3.A2.A. Intentionally Blank	2.3.11.A. Intentionally Blank
Units and Tools of Measurement	2.3.8.B. Develop strategies for determining areas and volumes of compound shapes and solids.	2.3.A1.B. Intentionally Blank	2.3.G.B. Intentionally Blank	2.3.A2.B. Intentionally Blank	2.3.11.B. Intentionally Blank
Calculations	2.3.8.C. Calculate volume, surface area, and degrees of angles; calculate circumference and area of circles, and use a measurement formula to solve for a missing quantity.	2.3.A1.C. Find missing quantities in measurement formulas by applying **equation** solving techniques.	2.3.G.C. Use properties of geometric figures and measurement formulas to solve for a missing quantity (e.g., the measure of a specific angle created by **parallel lines** and is a **transversal**).	2.3.A2.C. Solve a formula for a given **variable** using algebraic processes.	2.3.11.C. Use properties of geometric figures and measurement formulas to solve for a missing quantity (e.g., the measure of a specific angle created by **parallel lines** and a **transversal**).

2.3. Measurement and Estimation					
	2.3.8. GRADE 8	**2.3.A1. ALGEBRA I**	**2.3.G. GEOMETRY**	**2.3.A2. ALGEBRA II**	**2.3.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Conversions	2.3.8.D. Perform conversions within the **metric system** and within the **customary system,** including scale measurements between units of time and between units of temperature.	2.3.A1.D. Intentionally Blank	2.3.G.D. Intentionally Blank	2.3.A2.D. Intentionally Blank	2.3.11.D. Intentionally Blank
Relations	2.3.8.E. Describe how a change in linear dimension of an object affects its perimeter, area, and volume.	2.3.A1.E. Describe how a change in the value of one **variable** in a formula that utilizes linear variables affects the value of the measurement. M11.B.2.3.1	2.3.G.E. Describe how a change in the value of one **variable** in area and volume formulas affect the value of the measurement.	2.3.A2.E. Describe how a change in the value of one **variable** in formulas affects the value of the measurement.	2.3.11.E. Describe how a change in the value of one **variable** in a formula affects the value of the measurement.
Measurement Estimation	2.3.8.F. Estimate and verify measurements of rate and mass.	2.3.A1.F. Intentionally Blank	2.3.G.F. Intentionally Blank	2.3.A2.F. Intentionally Blank	2.3.11.F. Intentionally Blank

2.4. Mathematical Reasoning and Connections					
	2.4.8. GRADE 8	**2.4.A1. ALGEBRA I**	**2.4.G. GEOMETRY**	**2.4.A2. ALGEBRA II**	**2.4.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Reasoning	2.4.8.A. Draw **inductive** and **deductive** conclusions within mathematical contexts.	2.4.A1.A. Demonstrate the capability of justifying any step in an **equation** solving process by citing an algebraic property	2.4.G.A. Write formal proofs (**direct proofs, indirect proofs/proofs by contradiction**, use of counter-examples, truth tables, etc.) to validate **conjectures** or arguments.	2.4.A2.A. Intentionally Blank	2.4.11.A. Write formal proofs (**direct proofs, indirect proofs/proofs by contradiction**, use of counter-examples, truth tables, etc.) to validate **conjectures** or arguments.
Connections	2.4.8.B. Use *if…then* statements to construct simple **valid arguments**.	2.4.A1.B. Use *if…then* format to describe properties and theorems in algebra.	2.4.G.B. Use statements, **converses**, **inverses,** and **contrapositives** to construct **valid arguments** or to validate arguments relating to geometric theorems.	2.4.A2.B. Intentionally Blank	2.4.11.B. Use statements, **converses**, **inverses** and **contrapositives** to construct **valid arguments** or to validate arguments.

2.5. Mathematical Problem Solving and Communication					
	2.5.8. GRADE 8	**2.5.A1. ALGEBRA I**	**2.5.G. GEOMETRY**	**2.5.A2. ALGEBRA II**	**2.5.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Problem Solving	2.5.8.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, apply estimation skills as appropriate, check whether the plan makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.A1.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.G.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.A2.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.11.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.
Communication	2.5.8.B. Use precise mathematical language, notation and representations, including numerical tables and **equations**, simple algebraic **equations** and formulas, charts, graphs, and diagrams to explain and interpret results.	2.5.A1.B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing, and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas, and results.	2.5.G.B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing, and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas, and results.	2.5.A2.B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas, and results.	2.5.11.B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas, and results.

2.6. Statistics and Data Analysis					
	2.6.8. GRADE 8	**2.6.A1. ALGEBRA I**	**2.6.G. GEOMETRY**	**2.6.A2. ALGEBRA II**	**2.6.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Collection of Data	2.6.8.A. Understand and apply sampling techniques to gather data including simple random sampling and convenience sampling.	2.6.A1.A. Design and conduct an experiment using random sampling.	2.6.G.A. Intentionally Blank	2.6.A2.A. Intentionally Blank	2.6.11.A. Design and conduct an experiment using random sampling.
Organization and Display of Data	2.6.8.B. Organize and display one-**variable** data using appropriate data display, such as **stem-and-leaf** and **box-and-whisker plots**, and two **variable** data with **scatterplots.**	2.6.A1.B. Intentionally Blank	2.6.G.B. Intentionally Blank	2.6.A2.B. Intentionally Blank	2.6.11.B. Intentionally Blank
Numerical Summaries	2.6.8.C. Calculate **quartiles** for one-**variable** data and describe the **correlation** coefficient for two-**variable** data displayed in a **scatterplot**.	2.6.A1.C. Select or calculate the appropriate measure of **central tendency**, calculate and apply the **interquartile range** for one-**variable** data, and construct a **line of best fit** and calculate its **equation** for two-**variable** data.	2.6.G.C. Intentionally Blank	2.6.A2.C. Construct a **line of best fit** and calculate its **equation** for linear and non linear two-**variable** data.	2.6.11.C. Select or calculate the appropriate measure of **central tendency**, calculate and apply the **interquartile range** for one-**variable** data, and construct a **line of best fit** and calculate its **equation** for two-**variable** data.

2.6. Statistics and Data Analysis					
	2.6.8. GRADE 8	**2.6.A1. ALGEBRA I**	**2.6.G. GEOMETRY**	**2.6.A2. ALGEBRA II**	**2.6.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Statistical Comparisons	2.6.8.D. Compare data sets graphically using double-bar and double-line graphs and numerically using **mean**, **median**, **mode**, **range,** and **quartiles**.	2.6.A1.D. Intentionally Blank	2.6.G.D. Intentionally Blank	2.6.A2.D. Intentionally Blank	2.6.11.D. Intentionally Blank
Interpretation of Data	2.6.8.E. Determine the effect of extreme values on numerical summaries and calculate estimates based on survey results or graphs.	2.6.A1.E. Make predictions based on **lines of best fit** or draw conclusions on the value of a **variable** in a population based on the results of a sample.	2.6.G.E. Intentionally Blank	2.6.A2.E. Make predictions based on **lines of best fit** or draw conclusions on the value of a **variable** in a population based on the results of a sample.	2.6.11.E. Make predictions based on **lines of best fit** or draw conclusions on the value of a **variable** in a population based on the results of a sample.

2.7. Probability and Predictions					
	2.7.8. GRADE 8	**2.7.A1. ALGEBRA I**	**2.7.G. GEOMETRY**	**2.7.A2. ALGEBRA II**	**2.7.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Calculate Probabilities	2.7.8.A. Calculate the **probability** of an event involving *"and,"* *"or,"* or *"not"*.	2.7.A1.A. Calculate probabilities for independent, dependent, or compound events.	2.7.G.A. Use geometric figures and the concept of area to calculate probability.	2.7.A2.A. Use **probability** to predict the likelihood of an outcome in an experiment.	2.7.11.A. Use **probability** to predict the likelihood of an outcome in an experiment.
Prediction of Outcomes	2.7.8.B. Intentionally Blank	2.7.A1.B. Intentionally Blank	2.7.G.B. Intentionally Blank	2.7.A2.B. Intentionally Blank	2.7.11.B. Intentionally Blank
Representations of Probabilities	2.7.8.C. Determine the number of **combinations** and **permutations** for an event.	2.7.A1.C. Intentionally Blank	2.7.G.C. Intentionally Blank	2.7.A2.C. Compare odds and **probability**.	2.7.11.C. Compare odds and **probability**.
Display Simple Spaces	2.7.8.D. Intentionally Blank	2.7.A1.D. Intentionally Blank	2.7.G.D. Intentionally Blank	2.7.A2.D. Intentionally Blank	2.7.11.D. Intentionally Blank

2.7. Probability and Predictions				
2.7.8. GRADE 8	**2.7.A1. ALGEBRA I**	**2.7.G. GEOMETRY**	**2.7.A2. ALGEBRA II**	**2.7.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:				
Compare Theoretical and Experimental Probabilities	2.7.8.E. Find the experimental or theoretical **probability** of the outcomes of a simple or **compound event**.	2.7.A1.E. Intentionally Blank	2.7.G.E. Intentionally Blank	2.7.A2.E. Use **probability** to make judgments about the likelihood of various outcomes.	2.7.11.E. Use **probability** to make judgments about the likelihood of various outcomes.

2.8. Algebra and Functions

	2.8.8. GRADE 8	2.8.A1. ALGEBRA I	2.8.G. GEOMETRY	2.8.A2. ALGEBRA II	2.8.11. GRADE 11
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Algebraic Properties	2.8.8.A. Use the concept of equality to demonstrate understanding of the **inverse** properties of numbers and the addition and multiplication properties of equality.	2.8.A1.A. Intentionally Blank	2.8.G.A. Intentionally Blank	2.8.A2.A. Intentionally Blank	2.8.11.A. Intentionally Blank
Algebraic Manipulations	2.8.8.B. Evaluate and simplify algebraic **expressions** and solve and graph linear **equations** and **inequalities**.	2.8.A1.B. Evaluate and simplify not understood algebraic **expressions**, for example: sums of polynomials, products/quotients of exponential terms and product of a binomial times a trinomial; and solve and graph linear **equations** and **inequalities**.	2.8.G.B. Use algebraic representations to solve problems using coordinate geometry.	2.8.A2.B. Evaluate and simplify algebraic expressions, for example: products/quotients of polynomials, logarithmic expressions and complex fractions; and solve and graph**, quadratic, exponential,** and **logarithmic equations**; and, solve and graph **systems of equations** and **inequalities**.	2.8.11.B. Evaluate and simplify algebraic **expressions** and solve and graph linear, **quadratic**, **exponential,** and **logarithmic equations** and **inequalities**, and solve and graph **systems of equations** and **inequalities**.
Patterns	2.8.8.C. Find the missing elements and recognize, describe, and extend **patterns** to include **linear**, **exponential**, and simple **quadratic equations**.	2.8.A1.C. Identify and represent **patterns** algebraically and/or graphically.	2.8.G.C. Intentionally Blank	2.8.A2.C. Recognize, describe and generalize **patterns** using **sequences** and **series** to predict long-term outcomes	2.8.11.C. Recognize, describe and generalize patterns using sequences and series to predict long-term outcomes.

2.8. Algebra and Functions					
	2.8.8. GRADE 8	**2.8.A1. ALGEBRA I**	**2.8.G. GEOMETRY**	**2.8.A2. ALGEBRA II**	**2.8.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Functions	2.8.8.D. Create a table or graph from a functional rule.	2.8.A1.D. Demonstrate an understanding and apply properties of functions (**domain**, **range**) and characteristics of **linear function**s.	2.8.G.D. Intentionally Blank	2.8.A2.D. Demonstrate an understanding and apply properties of functions (**domain**, **range, inverses**) and characteristics of families of functions (**linear**, **polynomial**, **rational, exponential, logarithmic**).	2.8.11.D. Demonstrate an understanding and apply properties of functions (**domain**, range, **inverses**) and characteristics of families of functions (**linear**, **polynomial**, **ratio**nal, **trigonometric, exponential, logarithmic**).
Modeling	2.8.8.E. Use combinations of symbols and numbers to create **expressions**, **equations** in one or two **variables**, and **inequalities** in one **variable** that model problem situations.	2.8.A1.E. Use **combination**s of symbols and numbers to create **expressions, equations,** and **inequalities** in two or more **variable**s, **systems of equations,** and **inequalities**, and functional relationships that model problem situations.	2.8.G.E. Intentionally Blank	2.8.A2.E. Use **combination**s of symbols and numbers to create **expressions, equations,** and **inequalities** in two or more **variable**s, **systems of equations** and **inequalities**, and functional relationships that model problem situations.	2.8.11.E. Use combinations of symbols and numbers to create **expressions, equations,** and **inequalities** in two or more **variables**, **systems of equations** and **inequalities**, and functional relationships that model problem situations.
Interpret Results of Modeling	2.8.8.F. Interpret the results of solving **equations** in one or two **variable**s and **inequalities** in one **variable** in the context of the situation that motivated the model.	2.8.A1.F. Interpret the results of solving **equations, inequalities, systems of equations,** and systems of **inequalities** in the context of the situation that motivated the model.	2.8.G.F. Intentionally Blank	2.8.A2.F. Interpret the results of solving **equations, inequalities, systems of equations,** and systems of **inequalities** in the context of the situation that motivated the model.	2.8.11.F. Interpret the results of solving **equations, inequalities, systems of equations,** and **inequalities** in the context of the situation that motivated the model.

2.9. Geometry					
	2.9.8. GRADE 8	**2.9.A1. ALGEBRA I**	**2.9.G. GEOMETRY**	**2.9.A2. ALGEBRA II**	**2.9.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Definitions, Properties, and Relations	2.9.8.A. Name, describe and apply geometric relations for 1-dimensional shapes and 2-dimensional shapes and 3-dimensional solids.	2.9.A1.A. Use algebraic techniques to determine if two lines are **parallel** and **/** or **perpendicular**.	2.9.G.A. Identify and use properties and relations of geometric figures; create justifications for arguments related to geometric relations.	2.9.A2.A. Use algebraic techniques to determine if two lines are **parallel** and / or **perpendicular**; find points of intersections and distances between points.	2.9.11.A. Create justifications for arguments related to geometric relations.
Transformations and Symmetry	2.9.8.B. Predict and describe the result of a **translation** (slide), **rotation** (turn), or **reflection** (flip) of a 3-dimensional shape.	2.9.A1.B. Intentionally Blank	2.9.G.B. Use arguments based on **transformations** to establish **congruence** or **similarity** of 2- dimensional shapes.	2.9.A2.B. Intentionally Blank	2.9.11.B. Use arguments based on **transformations** to establish **congruence** or **similarity** of 2-dimensional shapes.
Coordinate Geometry	2.9.8.C. Plot ordered pairs and 2-dimensional shapes that satisfy given conditions on a 2-dimensional **coordinate system**.	2.9.A1.C. Use techniques from coordinate geometry to establish properties of lines and 2-dimensional shapes and solids.	2.9.G.C. Use techniques from coordinate geometry to establish properties of lines, 2-dimensional shapes.	2.9.A2.C. Use techniques from coordinate geometry to establish properties of lines, 2-dimensional shapes and solids.	2.9.11.C. Use techniques from coordinate geometry to establish properties of lines, shapes, and solids.

2.10. Trigonometry				
2.10.8. GRADE 8	**2.10.A1. ALGEBRA I**	**2.10.G. GEOMETRY**	**2.10.A2. ALGEBRA II**	**2.10.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:				
Right Triangle Concepts and Applications	2.10.8.A. Compute measures of sides and angles using **proportions**, the Pythagorean Theorem and right triangle relationships.	2.10.A1.A. Solve problems involving from the Pythagorean Theorem.	2.10.G.A. Identify, create, and solve practical problems involving right triangles using the **trigonometric ratios** and the Pythagorean Theorem.	2.10.A2.A. Intentionally Blank	2.10.11.A. Identify, create, and solve practical problems involving right triangles using the **trigonometric functions** and the Pythagorean Theorem.
Trigonometric Functions	2.10.8.B. Intentionally Blank	2.10.A1.B. Intentionally Blank	2.10.G.B. Intentionally Blank	2.10.A2.B. Intentionally Blank	2.10.11.B. Graph **periodic and circular functions;** describe properties of the graphs.

2.11. Concepts of Calculus					
	2.11.8. GRADE 8	**2.11.A1. ALGEBRA I**	**2.11.G. GEOMETRY**	**2.11.A2. ALGEBRA II**	**2.11.11. GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:					
Extreme Values	2.11.8.A. Analyze graphs of related quantities for minimum and maximum values and justify the findings.	2.11.A1.A. Intentionally Blank	2.11.G.A. Find the measures of the sides of a **polygon** with a given perimeter that will maximize the area of the polygon.	2.11.A2.A. Determine and interpret **maximum** and **minimum** values of a **function** over a specified interval.	2.11.11.A. Determine and interpret **maximum** and **minimum** values of a function over a specified interval.
Rates	2.11.8.B. Describe the concept of unit rate, **ratio,** and **slope** in the context of rate of change.	2.11.A1.B. Describe rates of change as modeled by linear **equation**s.	2.11.G.B. Intentionally Blank	2.11.A2.B. Analyze and interpret rates of growth/decay.	2.11.11.B. Analyze and interpret rates of growth/decay.
Accumulation of Areas and Volumes	2.11.8.C. Intentionally Blank	2.11.A1.C. Intentionally Blank	2.11.G.C. Use sums of areas of standard shapes to estimate the areas of complex shapes.	2.11.A2.C. Estimate areas under curves using sums of areas.	2.11.11.C. Estimate areas under curves using sums of areas.

VI. GLOSSARY

Absolute Value: A number's distance from zero on a number line. The absolute value of 2 is equal to the absolute value of -2.

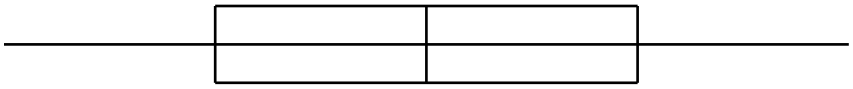
Algorithm: A method of performing an arithmetic operation.

Analog Time: Time displayed on a timepiece having hour and minute hands.

Array: Arrangement of a series of items according to the values of the items (e.g., largest to smallest).

Associative Property: A property of addition or multiplication in which the regrouping of the addends or factors does not change the outcome of the operations [i.e., (a + b) + c = a + (b + c) and (ab)c = a(bc)].

Box-and-Whisker Plot: A graphic method for showing a summary of data using median, quartiles and extremes of data.



Central Tendency: The degree of clustering of the values of a statistical distribution that is usually measured by the arithmetic mean, mode, or median.

Combination: A subset of the elements in a given set, without regard to the order in which those elements are arranged.

Commutative Property: A property of addition or multiplication in which the sum or product stays the same when the order of the addends or factors is changed (i.e., a + b = b + a and ab = ba).

Complementary Event: The opposite of an event. That is, the set of all outcomes of an experiment that are not included in an event. $P(A') = 1 - P(A)$.

Complex Numbers: Any number, real or imaginary, of the form *a + bi, where a* and *b* are real numbers and *I = -l.*

Composite Number: Any positive integer exactly divisible by one or more positive integers other than itself and 1.

Compound Event: An event that consists of two or more simple events; for example: A or B; A and. B and C.

Congruent: Having the same shape and the same size.

Conjecture: A statement believed to be true but not proved.

Contrapositive: The implication which results from replacing the antecedent by the negation of the consequent and the consequent by the negation of the antecedent.

Converse: The theorem (or implication) resulting from interchanging the hypothesis and conclusion.

Coordinate System: A method of locating points in the plane or in space by means of numbers. A point in the plane is located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.

Correlation : A measure of the mutual relationship between two variables.

Customary System: A system of weights and measures frequently used in the United States. The basic unit of weight is the pound; the basic unit of capacity is the quart.

Deductive Reasoning: The process of reasoning from statements accepted as true to reach a conclusion.

Dependent Event: The probability of one event depends on the outcome of another event.

Direct Proof: Uses an argument that makes direct use of the hypothesis and arrives at the conclusion.

Direct Variation: Two variables are so related that their ratio remains constant.

Distributive Property: Multiplication is distributive over addition and subtraction [i.e., a(b + c) = ab + ac and a(b-c) = ab-ac].

Domain: The set of all possible values for the unknown in an open sentence.

Equation: A statement of equality between two mathematical expressions (e.g., X + 5 = Y - 2).

Equivalent Forms: Different forms of numbers that name the same number (e.g., fraction, decimal, percent as $\frac{1}{2}$, .5, 50%).

Expanded Notation: Involves writing the number in expanded form to show the value of each digit (e.g., 15,629 = 10,000 + 5,000 + 600 + 20 + 9).

Exponent: A numeral used to tell how many times a number or variable is used as a factor (e.g., a^2, 2^n, y^x).

Exponential Function: A function whose general equation is $y = a \times b^x$ or $y = a \times b^{kx}$, where a, b and k stand for constants.

Exponential Notation: A way of writing numbers using bases and exponents [e.g., $425 = (4 \times 10^2) + (2 \times 10^1) + (5 \times 10^0)$].

Expression: A mathematical phrase that can include operations, numerals and variables. In algebraic terms: $2l + 3x$; in numeric terms: 13.4 - 4.7.

Factor: The number or variable multiplied in a multiplication expression.

Factorial: The expression n! (n factorial) is the product of all the numbers from 1 to n for any positive integer n.

Function: A relation in which each value of an independent variable is associated with a unique value of the dependent value.

Geoboard: A board with pegs aligned in grid fashion that permits rubber bands to be wrapped around pegs to form geometric figures.

Graphing Calculator: A calculator that will store and draw the graphs of several functions at once.

Histogram: A graphical display that shows frequencies as bars. It shows what proportion of cases fall into each of several categories.

Identity Property: In addition, adding 0 to a number leaves the number unchanged, or identical (2+0=2). Hence, zero is the identity element for addition. In multiplication, the product of any number and 1 is that number (4X1=4). Hence, one is the identity element for multiplication.

Independent Events: Events such that the outcome of the first event has no effect on the probabilities of the outcome of the second event (e.g., two tosses of the same coin are independent events).

Inductive Reasoning: Generalizations made from particular observations in a common occurrence.

Inequality: A mathematical sentence that contains a symbol (e.g., $>$, $<$, $\geq$, $\leq$ or $\neq$) in which the terms on either side of the symbol are unequal (e.g., $x < y$, $7 > 3$, $n \geq 4$).

Infinite: Has no end or goes on forever.

Integer: A number that is a positive whole number, a negative whole number or zero.

Interquartile Range: The inter-quartile range is a measure of the spread of or dispersion within a data set. It is calculated by taking the difference between the upper and the lower quartiles.

Inverse: A new conditional formed by negating both the antecedent and the consequent of a conditional.

Inverse Function: Formally, suppose f is a one-to-one function with a domain *A* and range *B*. The inverse function f^{-1} is a function with these properties:

- f^{-1} has domain *B* and range *A*.
- For all x in *B*, $f(f^{-1})(x)) = x$, or $f \circ f^{-1} = x$

Informally, if f is a function from A to B, then an inverse for f is a function from B to A, with the property that a round trip (a composition) from A to B to A returns each element to itself.

Inverse Operations: Operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations).

Inverse Variation: When the ratio of one variable to the reciprocal of the other is constant, one of them is said to vary inversely as the other.

Inverse Statement: Formed by negating the hypothesis and negating the conclusion of the original statement. *If p, then q* becomes *if not p, then not q*.

Irrational Number: A number that cannot be written as a simple fraction. It is an infinite and non-repeating decimal.

Limit: A number to which the terms of a sequence get closer so that beyond a certain term all terms are as close as desired to that number.

Line of Best Fit: The line that fits a set of data points with the smallest value for the sum of the squares of the errors (vertical distances) from the data points to the line; the regression line.

Linear Function: A function whose general equation is y = mx + b, where m and b stand for constants and $m \neq 0$.

Linear Measurement: Measurement in a straight line.

Logarithm: The exponent indicating the power to which a fixed number, the base, must be raised to produce a given number. For example, if $n^x = a$, the logarithm of a, with n as the base, is x; symbolically, $\log_n a = x$. If the base is 10, the log of 100 is 2.

Logarithmic Function: Any function in which an independent variable appears as the argument in the form of a logarithm; they are the inverse functions of exponential functions.

Manipulatives: Materials that allow students to explore mathematical concepts in a concrete mode.

Mathematical Model: A representation in the mathematical world of some phenomenon in the real world. It frequently consists of a function or relation specifying how two variables are related.

Matrix: A rectangular array of numbers representing such things as the coefficients in a system of equations arranged in rows and columns.

Maximum: The greatest number in a set of data.

Mean: The sum of the set of numbers divided by n, the number of numbers in the set.

Median: The number that lies in the middle when a set of numbers is arranged in order. If there are two middle values, the median is the mean of these values.

Metric System: A system of measurement used throughout the world based on factors of 10. It includes measures of length, weight and capacity.

Minimum: The least number in a set of data.

Missing Addend: A member of an addition number sentence in which that term is missing (e.g., 5 + __ = 8).

Mode: The number(s) that occurs most often in a set of numbers (e.g., in the set 1, 2, 3, 3, 5, 8; the mode is 3).

Models: Models are representations of real work phenomena. Numerical tables, graphs, equations or systems of equations are all mathematical representations that may be used to model real world phenomena. A good model should capture the essential character of whatever is being modeled.

Multiple: A number that is the product of a given integer and another integer (e.g., 6 and 9 are multiples of 3).

Normal Curve: A graphical plot of a mathematical function (frequency distribution) which is unimodal and symmetrical.

***One-to-one* Correspondence:** When one and only one element of a second set is assigned to an element of a first set, all elements of the second set are assigned, and every element of the first set has an assignment, the mapping is called one-to-one (e.g., in the set Bill Clinton, George Bush, Ronald Reagan, Jimmy Carter, Hillary Clinton, Barbara Bush, Nancy Reagan and Rosalynn Carter, there is a one-to-one correspondence between the pairs).

Open Sentence: A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown (e.g., $x + 5 = 9$, $y - 2 = 7$).

Order of Operations: Rules for evaluating an expression: work first within parentheses; then calculate all powers, from left to right; then do multiplications or divisions, from left to right; then do additions and subtractions, from left to right.

Patterns: Regularities in situations such as those in nature, events, shapes, designs and sets of numbers (e.g., spirals on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12,…).

Permutation: An arrangement of a given number of objects from a given set in which the order of the objects is significant.

Perpendicular Lines: Two lines that intersect to form right angles (e.g., ⊥, ↵, ⌜).

Plotting Points: Locating points by means of coordinates, or a curve by plotted points, representing an equation by means of a curve so constructed.

Polygon: A union of segments connected end to end, such that each segment intersects exactly two others at its endpoints.

Powers: A number expressed using an exponent. The number 5^3 is read five to the third power or five cubed.

Prime: An integer greater than one whose only positive **f**actors are 1 and itself (e.g., 2, 3, 5, 7, 11, 13, 17, and 19).

Probability: A number from 0 to 1 that indicates how likely something is to happen.

Problem-Solving: Finding ways to reach a goal when no routine path is apparent.

Proof by Contradiction: A proof in which, if s is to be proven, one reasons from not**-**s until a contradiction is deduced; from this it is concluded that not**-**s is false, which means that s is true.

Proportion: An equation of the form $\frac{a}{b} = \frac{c}{d}$ that states that the two ratios are equivalent.

Quadrilateral: A four-sided polygon.

Quartiles: The three values that divide an ordered set into four subsets of approximately equal size. The second quartile is the median.

Radian: A unit of angular measure equal to $\frac{1}{2\pi}$ of a complete revolution.

Range (1): The difference between the greatest number and the least number in a set of data.

Range (2): The set of output values for a function.

Rate of Change: The limit of the ratio of an increment of the function value at the point to that of the independent variable as the increment of the variable approaches zero.

Ratio: A comparison of two numbers by division.

Rational numbers: Any number that can be written in the form $\frac{a}{b}$ where a is any integer and b is any integer except zero.

Real Numbers: The set consisting of all rational numbers and all irrational numbers.

Reasonableness: Quality of a solution such that it is not extreme or excessive.

Reciprocal: The fractional number that results from dividing one by the number.

Rectangular Prism: A three-dimensional figure whose sides are all rectangles; a box.

Reflection: A transformation that produces the mirror image of a geometric figure.

Regression: The line that represents the least deviation from the points in a scatter plot of data.

Regular Polygon: A polygon in which all sides have the same measure and all angles have the same measure.

Relation: A set of ordered pairs.

Reliability: The extent to which a measuring procedure yields the same results on repeated trials.

Repeated Addition: A model for multiplication (e.g., $2 + 2 + 2 = 3 \times 2$).

Rotation: A transformation that maps every point in the plane to its image by rotating the plane around a fixed point or line.

Scatterplot: A graph of plotted points that show the relationship between two sets of data.

Scientific Calculator: A calculator that represents very large or very small numbers in scientific notation with the powering, factorial, square root, negative and reciprocal keys.

Scientific Notation: A way in writing a number of terms of an integer power of 10 multiplied by a number greater than or equal to 1 and less than 10.

Sequence: A set of ordered quantities (e.g., positive integers).

Series: The indicated sum of the terms of a sequence.

Similarity: Having the same shape but not necessarily the same size.

Simple Event: An event whose probability can be obtained from consideration of a single occurrence (e.g., the tossing of a coin is a simple event).

Simulation: Modeling a real event without actually observing the event.

Slope: The slope of a line is the ratio of the change in y to the corresponding change in x; the constant m in the linear function equation; rise/run.

Standard Deviation: The square root of the variance.

Stem-and-Leaf Plot: A frequency distribution made by arranging data (e.g., student scores on a test were 98, 96, 85, 93, 83, 87, 85, 87, 93, 75, 77 and 83). This data is displayed in a stem-and-leaf plot below.

9	8, 6, 3, 3
8	7, 7, 5, 5, 3, 3
7	7, 5

Systems of Equations: Two or more equations that are conditions imposed simultaneously on all the variables, but may or may not have common solutions (e.g., $x + y = 2$, and $3x + 2y = 5$).

Symmetry: A line of symmetry separates a figure into two congruent halves, each of which is a reflection of the other (e.g., ∅, the line through the center of the circle divides it into congruent halves).

T-Test: A statistical test done to test the difference of means of two samples.

Tessellation: A repetitive pattern of polygons that covers an area with no holes and no overlaps (e.g., floor tiles).

Transformation: An operation on a geometric figure by which each point gives rise to a unique image.

Translation: A transformation that moves a geometric figure by sliding each of the points the same distance in the same direction.

Tree Diagram: A diagram used to show the total number of possible outcomes in a probability experiment.

Trigonometric Functions: A function (e.g., sine, cosine, tangent, cotangent, secant, cosecant) whose independent variable is an angle measure, usually in degrees or radians.

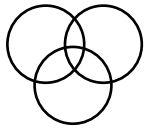
Valid Argument: An argument with the property that no matter what statements are substituted in the premises, the truth value of the form is true. If the premises are true, then the conclusion is true.

Variable: A symbol used to stand for any one of a given set of numbers or other objects (e.g., in the equation $y = x + 5$, y and x are variables).

Variance: In a data set, the sum of the squared deviations divided by one less than the number of elements in the set (sample variance s^2) or by the number of elements in the set (population variance σ^2).

Vector: A quantity that has both magnitude and direction (e.g., physical quantities such as velocity and force).

Venn Diagram: A display that pictures unions and intersections of sets.



Volume: The amount of space enclosed in a space (3-dimensional) figure, measured in cubic units.

Y-Intercept: The y-intercept of a line is the y-coordinate of the point at which the graph of an equation crosses the y-axis.

π: pi, the ratio of the circumference of a circle to its diameter: 3.1415926535.