

# Academic Standards for Mathematics

*Pre-K-3*  
*REVISED - May 18, 2010*



*Pennsylvania Department of Education*

*These standards are offered as a voluntary resource for Pennsylvania's schools and await action by the State Board of Education.*

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

IV. TABLE OF CONTENTS

**Introduction.....V.**

THE ACADEMIC STANDARDS

**Number, Number Systems and Number Relationships...2.1.**

- Count and Compare Numbers*
- Represent Numbers in Equivalent Forms*
- Concepts of Numbers and Relationships*
- Place Value*
- Number Theory*
- Concepts and Applications of Operations*

**Computation and Estimation.....2.2.**

- Fluency in Basic Facts*
- Computation*
- Evaluate Numerical Expressions*
- Numerical Estimation*

**Measurement and Estimation.....2.3.**

- Concept of Measurement*
- Units and Tools of Measurement*
- Calculations*
- Conversions*
- Relations*
- Measurement Estimation*

**Mathematical Reasoning and Connections.....2.4.**

- Reasoning*
- Connections*

**Mathematical Problem Solving and Communication.....2.5.**

- Problem Solving*
- Communication*

**Statistics and Data Analysis.....2.6.**

- Collection of Data*
- Organization and Display of Data*
- Numerical Summaries*
- Statistical Comparisons*
- Interpretation of Data*

**Probability and Predictions.....2.7.**

- Calculate Probabilities*
- Prediction of Outcomes*
- Representations of Probabilities*
- Display Simple Spaces*
- Compare Theoretical and Experimental Probabilities*

**Algebra and Functions.....2.8.**

- Algebraic Properties*
- Algebraic Manipulations*
- Patterns*
- Functions*
- Modeling*
- Interpret Results of Modeling*

**Geometry.....2.9.**

- Definitions, Properties, and Relations*
- Transformations and Symmetry*
- Coordinate Geometry*

**Trigonometry.....2.10.**

- Right Triangle Concepts and Applications*
- Trigonometric Functions*

**Concepts of Calculus.....2.11.**

- Extreme Values*
- Rates*
- Accumulation of Areas and Volumes*

**Glossary.....VI.**

# ELEMENTARY STANDARDS

## Grades Pre-K-3

### Mathematics

#### V. INTRODUCTION

This document includes Mathematics Standards:

- ◇ 2.1. Numbers, Number Systems, and Number Relationships
- ◇ 2.2. Computation and Estimation
- ◇ 2.3. Measurement and Estimation
- ◇ 2.4. Mathematical Reasoning and Connections
- ◇ 2.5. Mathematical Problem Solving and Communication
- ◇ 2.6. Statistics and Data Analysis
- ◇ 2.7. Probability and Predictions
- ◇ 2.8. Algebra and Functions
- ◇ 2.9. Geometry
- ◇ 2.10. Trigonometry
- ◇ 2.11. Concepts of Calculus

The Mathematics Standards describe what students should know and be able to do at grade levels. They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school.

With each standard divided into conceptual strands, this document avoids repetition of learned skills and makes an obvious progression across grade levels less explicit. Teachers shall expect that students know and can apply the concepts and skills expressed at the preceding level. Consequently, previous learning is reinforced but not re-taught.

Students who achieve these mathematical standards will be able to communicate mathematically. Although it is an interesting and enjoyable study for its own sake, mathematics is most appropriately used as a tool to help organize and understand information from other academic disciplines. Because our capacity to deal with all things mathematical is changing rapidly, students must be able to bring the most modern and effective technology to bear on their learning of mathematical concepts and skills.

A glossary is included to assist the reader in understanding terminology contained in the standards. Words in bold faced text are included in the glossary.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.1. Numbers, Number Systems and Number Relationships</b>					
<b>2.1.PK. GRADE Pre-K</b>		<b>2.1.K. GRADE K</b>	<b>2.1.1. GRADE 1</b>	<b>2.1.2. GRADE 2</b>	<b>2.1.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Count and Compare Numbers</b>	2.1.PK.A. Demonstrate the relationship between numbers and quantities, including rote counting, one-to-one correspondence up to 10 objects, and comparing values of whole numbers up to 10.	2.1.K.A. Demonstrate the relationship between numbers and quantities, including rote counting, one-to-one correspondence, and counting by tens, and comparing values of whole numbers up to 20.	2.1.1.A. Demonstrate the relationship between numbers and quantities, including place value, one-to-one correspondence, rote counting, counting by twos to 20, counting by tens and fives, and comparing values of whole numbers up to 100.	2.1.2.A. Demonstrate the relationship between numbers and quantities, including place value; one-to-one correspondence; rote counting; counting by twos, fives and tens; and comparing values of whole numbers up to 500.	2.1.3.A. Apply <b>one-to-one correspondence</b> and number <b>patterns</b> to count up and count back and to compare values of whole numbers and values of money.
<b>Represent Numbers in Equivalent Forms</b>	2.1.PK.B. Represent equivalent forms of the same number through the use of pictures and concrete objects up to 10.	2.1.K.B. Represent equivalent forms of the same number through the use of pictures and concrete objects (including penny, nickel, and dime), up to 20.	2.1.1.B. Represent equivalent forms of the same number through the use of pictures and concrete objects (including penny, nickel, dime, and quarter), up to 100.	2.1.2.B. Represent equivalent forms of the same number through the use of pictures and concrete objects (including penny, nickel, dime, quarter, and dollar), up to 500.	2.1.3.B. Represent <b>equivalent forms</b> of the same number through the use of concrete objects, drawings, word names, and symbols.
<b>Concepts of Numbers and Relationships</b>	2.1.PK.C. Use concrete objects, drawings, diagrams or models to combine, separate and name groups of objects.	2.1.K.C. Use concrete objects, drawings, diagrams or models to group objects into sets of ten; separate objects into equal parts.	2.1.1.C. Use concrete objects, drawings, diagrams or models to show the concept of a fraction as part of a whole; use whole numbers and fractions (halves and fourths) to represent quantities.	2.1.2.C. Use drawings or models to show the concept of a fraction as part of a whole; use whole numbers and simple fractions (halves, thirds, and fourths) to represent quantities.	2.1.3.C. Use drawings, diagrams or <b>models</b> to show the concept of fraction as part of a whole.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

2.1. Numbers, Number Systems and Number Relationships						
2.1.PK. GRADE Pre-K		2.1.K. GRADE K	2.1.1. GRADE 1	2.1.2. GRADE 2	2.1.3. GRADE 3	
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
Concepts and Applications of Operations	Place Value	Intentionally Blank	2.1.K.D. Use concrete objects to demonstrate regrouping ones to tens, with adult assistance.	2.1.1.D. Apply place value concepts and base-ten numeration to order and compare whole numbers up to 100.	2.1.2.D. Apply place value concepts and base-ten numeration to order and compare whole numbers up to 500.	2.1.3.D. Apply place value concepts and base-ten numeration to order and compare whole numbers.
	Number Theory	Intentionally Blank	2.1.K.E. Recognize even and odd number patterns.	2.1.1.E. Describe even and odd numbers as they relate to a number pattern.	2.1.2.E. Apply number <b>patterns</b> to represent numbers in various ways (skip counting , repeated addition/subtraction).	2.1.3.E. Apply number <b>patterns</b> even and odd, <b>factors</b> and <b>multiples</b> to represent numbers in various ways.
	2.1.PK.F. Solve oral word problems using concrete objects, with teacher assistance.	2.1.K.F. Use concrete objects to solve addition and subtraction word problems.	2.1.1.F. Select the appropriate operation (addition or subtraction) to solve problems.	2.1.2.F. Use the <b>inverse</b> relationships between addition and subtraction to determine unknown quantities to solve problems.	2.1.3.F. Understand the concepts of addition and subtraction and use the <b>inverse</b> relationships between addition and subtraction to determine unknown quantities in <b>equations</b> .	

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.2. Computation and Estimation</b>					
<b>2.2.PK. Grade Pre-K</b>		<b>2.2.K. GRADE K</b>	<b>2.2.1. GRADE 1</b>	<b>2.2.2. GRADE 2</b>	<b>2.2.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Fluency in Basic Facts</b>	2.2.PK.A. Intentionally Blank	2.2.K.A. Intentionally Blank	2.2.1.A. Apply concepts of addition and subtraction to solve problems up to ten.	2.2.2.A. Develop fluency in the use of basic facts for addition and subtraction	2.2.3.A. Develop <b>fluency</b> in the use of basic facts for the four operations.
<b>Computation</b>	2.2.PK.B. Represent and explain the results of adding and subtracting sets of objects up to and including six, using math vocabulary	2.2.K.B. Represent and explain the results of adding and subtracting sets of objects up to and including ten, using math vocabulary	2.2.1.B. Demonstrate strategies for addition and subtraction in order to solve single- and double-digit addition and subtraction problems.	2.2.2.B. Add and subtract single and double-digit numbers with and without regrouping, to include problems with money.	2.2.3.B. Add and subtract single- and double-digit numbers with regrouping and triple-digit numbers, without regrouping including problems with money.
<b>Evaluate Numerical Expressions</b>	2.2.PK.C. Intentionally Blank	2.2.K.C. Intentionally Blank	2.2.1.C. Intentionally Blank	2.2.2.C. Intentionally Blank	2.2.3.C. Intentionally Blank
<b>Numerical Estimation</b>	2.2.PK.D. Estimate how many objects are in a set/group up to and beyond ten objects and conclude the <b>reasonableness</b> of those estimates.	2.2.K.D. Estimate how many objects are in a set/group up to and including twenty objects and conclude the <b>reasonableness</b> of those estimates.	2.2.1.D. Estimate values and sums of quantities and conclude the <b>reasonableness</b> of those estimates.	2.2.2.D. Estimate values, sums, and differences of quantities and conclude the <b>reasonableness</b> of those estimates.	2.2.3.D. Estimate values, sums, and differences of quantities and conclude the <b>reasonableness</b> of those estimates.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.3. Measurement and Estimation</b>					
<b>2.3.PK. GRADE Pre- K</b>		<b>2.3.K. GRADE K</b>	<b>2.3.1 GRADE 1</b>	<b>2.3.2. GRADE 2</b>	<b>2.3.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Concept of Measurement</b>	2.3.PK.A. Identify characteristics that are measurable.	2.3.K.A. Identify similarities and differences in measurement of objects.	2.3.1.A. Demonstrate that a single object has attributes that can be measured.	2.3.2.A. Demonstrate that a single object has different attributes that can be measured in multiple ways.	2.3.3.A. Demonstrate an understanding of measurable characteristics and the need to quantify those characteristics.
<b>Units and Tools of Measurement</b>	2.3.PK.B. Use concrete objects as non-standard units to estimate and measure, with adult awareness.	2.3.K.B. Use concrete objects as non-standard units to estimate and measure.	2.3.1.B. Use concrete objects to measure length by repeating the number of nonstandard or standard units.	2.3.2.B. Use tools to estimate and measure in standard units.	2.3.3.B. Identify a measurable characteristic of an object, select an appropriate standard or non-standard unit of measure and tool, and determine the measurement to a specified level of accuracy.
<b>Calculations</b>	2.3.PK.C. Order up to three events based on time.	2.3.K.C. Order events based on time.	2.3.1.C. Tell time on an analog and digital clock to the nearest hour and half hour.	2.3.2.C. Tell time on an analog and digital clock to the nearest minute.	2.3.3.C. Tell time on an analog and digital clock, identify times of day and night as a.m. and p.m., and calculate elapsed time.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.3. Measurement and Estimation</b>						
<b>2.3.PK. GRADE Pre- K</b>		<b>2.3.K. GRADE K</b>		<b>2.3.1 GRADE 1</b>	<b>2.3.2. GRADE 2</b>	<b>2.3.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Conversions</b>	2.3.PK.D. Intentionally Blank	2.3.K.D. Intentionally Blank	2.3.1.D. Intentionally Blank	2.3.2.D. Intentionally Blank	2.3.3.D. Identify equivalent measurements within the same system.	
<b>Relations</b>	2.3.PK.E. Intentionally Blank	2.3.K.E. Intentionally Blank	2.3.1.E. Intentionally Blank	2.3.2.E. Intentionally Blank	2.3.3.E. Intentionally Blank	
<b>Measurement Estimation</b>	2.3.PK.F. Compare concrete objects to estimate and verify measurements of length.	2.3.K.F. Compare concrete objects to determine greater or lesser attributes (length, weight).	2.3.1.F. Compare concrete objects to determine greater or lesser attributes (length, weight, capacity).	2.3.2.F. Estimate and verify measurements of length, weight, and capacity.	2.3.3.F. Estimate and verify measurements of length, area, weight, and capacity.	



ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.4. Mathematical Reasoning and Connections</b>					
<b>2.4.PK. GRADE Pre-K</b>		<b>2.4.K. GRADE K</b>	<b>2.4.1. GRADE 1</b>	<b>2.4.2. GRADE 2</b>	<b>2.4.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Reasoning</b>	2.4.PK.A. Describe the process (es) (e.g., think aloud) related to problem solving situations.	2.4.K.A. Draw conclusions and describe thought process (es) (e.g., think aloud) related to problem solving situations.	2.4.1.A. Draw conclusions and describe thought process (es) (e.g., think aloud) related to problem solving situations.	2.4.2.A. Use <b>models</b> and number facts to draw conclusions and explain reasons for conclusions.	2.4.3.A. Use <b>models</b> and number facts to draw conclusions and explain reasons for conclusions.
<b>Connections</b>	2.4.PK.B. Intentionally Blank	2.4.K.B. Use terms of logic to describe models (e.g., <i>all, or, every, none, some, or many</i> ).	2.4.1.B. Use concrete objects to explain precise terms of logic (e.g., <i>all, or, every, none, some, or many</i> ).	2.4.2.B. Describe and/or interpret statements made with precise language of logic (e.g., <i>all, or, every, none, some, or many</i> .)	2.4.3.B. Interpret statements made with precise language of logic (e.g., <i>all, or, every, none, some, or many</i> .)

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.5. Mathematical Problem Solving and Communication</b>						
<b>2.5.PK. GRADE Pre-K</b>		<b>2.5.K. GRADE K</b>		<b>2.5.1 GRADE 1</b>	<b>2.5.2. GRADE 2</b>	<b>2.5.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>						
<b>Problem Solving</b>	2.5.PK.A. Solve a problem and discuss how it was solved in grade appropriate contexts..	2.5.K.A. Solve a problem and explain how it was solved in grade appropriate contexts.	2.5.1.A. Solve the problem and check if answer makes sense. Explain how problem is solved in grade appropriate contexts.	2.5.2.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	2.5.3.A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense, and explain how the problem was solved in grade appropriate contexts.	
	2.5.PK.B. Use appropriate mathematical vocabulary when explaining how to solve a problem.	2.5.K.B. Use appropriate mathematical vocabulary when explaining how to solve a problem.	2.5.1.B. Use appropriate mathematical vocabulary when explaining how to solve a problem.	2.5.2.B. Use appropriate mathematical vocabulary when explaining how to solve a problem.	2.5.3.B. Use appropriate mathematical vocabulary when explaining how to solve a problem.	

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.6. Statistics and Data Analysis</b>					
	<b>2.6.PK. GRADE Pre-K</b>	<b>2.6.K. GRADE K</b>	<b>2.6.1. GRADE 1</b>	<b>2.6.2. GRADE 2</b>	<b>2.6.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Collection of Data</b>	2.6.PK.A. Use environmental objects for data collection purposes	2.6.K.A. Gather data in response to questions posed to learners.	2.6.1.A. Gather data from surveys and observations within the classroom or homes.	2.6.2.A. Gather data from surveys and observations within the classroom or homes.	2.6.3.A. Gather data from surveys and observations within the classroom or homes.
<b>Organization and Display of Data</b>	2.6.PK.B. Organize and display objects by one attribute.	2.6.K.B. Organize and display objects by one or more attributes.	2.6.1.B. Organize and display data using pictures, tallies, charts, bar graphs and pictographs.	2.6.2.B. Organize and display data using pictures, tallies, charts, bar graphs and pictographs.	2.6.3.B. Organize and display data using pictures, tallies, charts, bar graphs and pictographs.
<b>Numerical Summaries</b>	2.6.PK.C. Answer questions based on data shown on graphs or charts.	2.6.K.C. Answer questions based on data shown on graphs or charts.	2.6.1.C. Describe data displayed in a diagram, graph or table.	2.6.2.C. Describe data displayed in a diagram, graph or table.	2.6.3.C. Describe data displayed in a diagram ( <b>e.g., Venn</b> ) a graph or a table.
<b>Statistical Comparisons</b>	2.6.PK.D. Answer comparative questions based on representations of data.	2.6.K.D. Answer comparative questions based on representations of data.	2.6.1.D. Answer comparative questions based on representations of data.	2.6.2.D. Analyze representations of data and compare the data from two categories.	2.6.3.D. Analyze data shown in tables, charts, diagrams, and graphs; compare the data from two categories displayed in a graph and compare representations of a set of data in different graphs.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.6. Statistics and Data Analysis</b>					
<b>2.6.PK. GRADE Pre-K</b>		<b>2.6.K. GRADE K</b>	<b>2.6.1. GRADE 1</b>	<b>2.6.2. GRADE 2</b>	<b>2.6.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Interpretation of Data</b>	2.6.PK.E. Draw conclusions about information shown on a graph or chart.	2.6.K.E. Draw conclusions about information shown on a graph or chart.	2.6.1.E. Draw conclusions and identify patterns based on a comparison to data displayed in a graph.	2.6.2.E. Identify patterns and predict trends based on a comparison to data displayed in a graph.	2.6.3.E. Determine the <b>reasonableness</b> of a statement based on a comparison to data displayed in a graph.

<b>2.7. Probability and Predictions</b>					
<b>2.7.PK. GRADE Pre-K</b>		<b>2.7.K. GRADE K</b>	<b>2.7.1. GRADE 1</b>	<b>2.7.2. GRADE 2</b>	<b>2.7.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Calculation of Probabilities</b>	2.7.PK.A. Intentionally Blank	2.7.K.A. Determine the probability of an event occurring.	2.7.1.A. Determine the probability of an event occurring.	2.7.2.A. Determine the likelihood of an event occurring using words like certain, likely, unlikely or impossible.	2.7.3.A. Determine the chance of an event occurring by performing <b>simulations</b> with concrete devices (e.g., dice, spinner).
<b>Prediction of Outcomes</b>	2.7.PK.B. Intentionally Blank	2.7.K.B. Predict outcomes of events.	2.7.1.B. Predict and explain the outcomes of events.	2.7.2.B. Predict if different outcomes of the same event are equally likely or not equally likely.	2.7.3.B. Determine whether different outcomes of the same event are equally likely or not equally likely.

**ELEMENTARY STANDARDS**  
**Grades Pre-K-3**  
**Mathematics**

<b>2.7. Probability and Predictions</b>											
<b>2.7.PK. GRADE Pre-K</b>		<b>2.7.K. GRADE K</b>		<b>2.7.1. GRADE 1</b>		<b>2.7.2. GRADE 2</b>		<b>2.7.3. GRADE 3</b>			
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>											
<b>Representations of Probabilities</b>	2.7.PK.C. Intentionally Blank		2.7.K.C. Intentionally Blank		2.7.1.C. Intentionally Blank		2.7.2.C. Write the likelihood of an event as a simple fraction (e.g., $\frac{1}{2}$ ).		2.7.3.C. Write the likelihood of an event as a fraction.		
	<b>Display Sample Spaces</b>	2.7.PK.D. List or graph the possible results of an experiment, with adult assistance.		2.7.K.D. List or graph the possible results of an experiment, with adult assistance.		2.7.1.D. List or graph the possible results of an experiment.		2.7.2.D. List or graph the possible results of an experiment.		2.7.3.D. List or graph the possible results of an experiment.	
		<b>Compare Theoretical and Experimental Probabilities</b>	2.7.PK.E. Answer questions about an observed event.		2.7.K.E. Answer questions about predictions and actual outcomes based on data.		2.7.1.E. Answer questions about predictions and actual outcomes based on data.		2.7.2.E. Ask and answer questions about predictions and actual outcomes based on data.		2.7.3.E. Determine that there can be a difference between predicted and actual outcomes.

**ELEMENTARY STANDARDS**  
**Grades Pre-K-3**  
**Mathematics**

<b>2.8. Algebra and Functions</b>					
<b>2.8.PK. GRADE Pre-K</b>		<b>2.8.K. GRADE K</b>	<b>2.8.1. GRADE 1</b>	<b>2.8.2. GRADE 2</b>	<b>2.8.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Algebraic Properties</b>	2.8.PK.A. Use concrete objects to demonstrate understanding of equality.	2.8.K.A. Use concrete objects to demonstrate understanding of equal and not equal.	2.8.1.A. Use the concept of equality and concrete objects to demonstrate understanding of the commutative and associative properties.	2.8.2.A. Use the concept of equality and concrete objects to demonstrate understanding of the commutative and associative properties.	2.8.3.A. Use the concept of equality and concrete objects to demonstrate understanding of commutative, associative, and identity properties.
<b>Algebraic Manipulations</b>	2.8.PK.B. Recreate and explain simple story problems using manipulatives, with adult assistance.	2.8.K.B. Recreate and explain simple story problems using manipulatives.	2.8.1.B. Use concrete objects and trial and error to solve number sentences.	2.8.2.B. Use concrete objects and trial and error to solve number sentences (equations and inequalities).	2.8.3.B. Use concrete objects and trial and error to solve number sentences (equations and inequalities).
<b>Patterns</b>	2.8.PK.C. Recognize and replicate number and geometric patterns.	2.8.K.C. Recognize, describe, extend, replicate and transfer number and geometric patterns.	2.8.1.C. Recognize, describe, extend, replicate and transfer number and geometric patterns.	2.8.2.C. Recognize, describe, extend, create, and replicate a variety of patterns including attribute, activity, number, and geometric patterns.	2.8.3.C. Recognize, describe, extend, create, and replicate a variety of patterns including attribute, activity, number, and geometric patterns.
<b>Functions</b>	2.8.PK.D. Intentionally Blank	2.8.K.D. Use a rule represented by concrete objects to find a missing value.	2.8.1.D. Use a rule to find a missing addend or symbol to make a number sentence true, with adult assistance.	2.8.2.D. Use a rule to find a missing element that makes an addition or subtraction number sentence true.	2.8.3.D. Use a rule to find a missing value and determine a rule for a given pattern.
<b>Modeling</b>	2.8.PK.E. Use concrete objects to represent mathematical situations.	2.8.K.E. Use concrete objects to represent mathematical situations.	2.8.1.E. Use concrete objects, symbols and numbers to represent mathematical situations.	2.8.2.E. Use concrete objects, symbols and numbers to represent mathematical situations.	2.8.3.E. Use concrete objects or combinations of symbols and numbers to represent expressions, equations, and inequalities that model mathematical situations.

**ELEMENTARY STANDARDS**  
**Grades Pre-K-3**  
**Mathematics**

<b>2.8. Algebra and Functions</b>					
<b>2.8.PK. GRADE Pre-K</b>		<b>2.8.K. GRADE K</b>	<b>2.8.1. GRADE 1</b>	<b>2.8.2. GRADE 2</b>	<b>2.8.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Interpret Results of Modeling</b>	2.8.PK.F. Describe data from classroom graphs and charts.	2.8.K.F. Describe data from classroom graphs and charts.	2.8.1.F. Describe and answer questions about data from classroom graphs and charts.	2.8.2.F. Describe, compare, and answer questions about data on classroom graphs and charts.	2.8.3.F. Describe data represented in a table, chart, or number sentence and/or create a story that matches that data.

<b>2.9. Geometry</b>					
<b>2.9.PK. GRADE Pre-K</b>		<b>2.9.K. GRADE K</b>	<b>2.9.1 GRADE 1</b>	<b>2.9.2 GRADE 2</b>	<b>2.9.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Definitions, Properties, and Relations</b>	2.9.PK.A. Sort common 2-dimensional shapes.	2.9.K.A. Identify and describe common 2-dimensional shapes.	2.9.1.A. Name, describe and draw/build 2-dimensional shapes	2.9.2.A. Name, describe and draw/build 2- and 3-dimensional shapes	2.9.3.A. Name, describe and draw/build 2- and 3-dimensional shapes

ELEMENTARY STANDARDS  
 Grades Pre-K-3  
 Mathematics

<b>2.9. Geometry</b>					
<b>2.9.PK. GRADE Pre-K</b>		<b>2.9.K. GRADE K</b>	<b>2.9.1 GRADE 1</b>	<b>2.9.2 GRADE 2</b>	<b>2.9.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Transformations and Symmetry</b>	2.9.PK.B. Intentionally Blank	2.9.K.B. Identify and draw lines of <b>symmetry</b> , with adult assistance.	2.9.1.B. Identify and draw lines of <b>symmetry</b> .	2.9.2.B. Identify and draw lines of <b>symmetry</b> .	2.9.3.B. Identify and draw lines of <b>symmetry</b> .
	2.9.PK.C. Intentionally Blank	2.9.K.C. Intentionally Blank	2.9.1.C. Intentionally Blank	2.9.2.C. Intentionally Blank	2.9.3.C. Identify locations of points with whole number coordinates on a number line or on a 2- dimensional <b>coordinate system</b> .
<b>Coordinate Geometry</b>					



ELEMENTARY STANDARDS  
 Grades Pre-K-3  
 Mathematics

<b>2.10. Trigonometry</b>					
<b>2.10.PK. GRADE Pre-K</b>		<b>2.10.K. GRADE K</b>	<b>2.10.1. GRADE 1</b>	<b>2.10.2. GRADE 2</b>	<b>2.10.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Right Triangle Concepts and Applications</b>	2.10.PK.A. Intentionally Blank	2.10.K.A. Intentionally Blank	2.10.1.A. Intentionally Blank	2.10.2.A. Intentionally Blank	2.10.3.A. Identify right angles in the environment.
<b>Trigonometric Functions</b>	2.10.PK.B Intentionally Blank	2.10.K.B Intentionally Blank	2.10.1.B. Intentionally Blank	2.10.2.B. Intentionally Blank	2.10.3.B. Intentionally Blank

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>2.11. Concepts of Calculus</b>					
<b>2.11.PK. GRADE Pre-K</b>		<b>2.11.K. GRADE K</b>	<b>2.11.1. GRADE 1</b>	<b>2.11.2. GRADE 2</b>	<b>2.11.3. GRADE 3</b>
<i>Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:</i>					
<b>Extreme Values</b>	2.11.PK.A. Intentionally Blank	2.11.K.A. Order whole numbers, 0 to 20, with least to greatest value.	2.11.1.A. Order whole numbers, 0 to 100, with least to greatest value.	2.11.2.A. Order whole numbers, 0 to 500, with least to greatest value.	2.11.3.A. Identify whole number quantities and measurements from least to most and greatest value.
<b>Rates</b>	2.11.PK.B. Intentionally Blank	2.11.K.B. Intentionally Blank	2.11.1.B. Intentionally Blank	2.11.2.B. Intentionally Blank	2.11.3.B. Intentionally Blank
<b>Accumulation of Areas and Volumes</b>	2.11.PK.C. Intentionally Blank	2.11.K.C. Intentionally Blank	2.11.1.C. Intentionally Blank	2.11.2.C. Intentionally Blank	2.11.3.C. Intentionally Blank

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics  
**VI. GLOSSARY**

**Absolute Value:** A number's distance from zero on a number line. The absolute value of 2 is equal to the absolute value of -2.

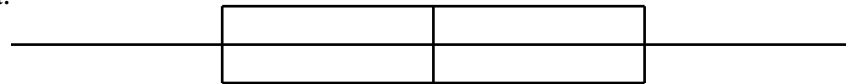
**Algorithm:** A method of performing an arithmetic operation.

**Analog Time:** Time displayed on a timepiece having hour and minute hands.

**Array:** Arrangement of a series of items according to the values of the items (e.g., largest to smallest).

**Associative Property:** A property of addition or multiplication in which the regrouping of the addends or factors does not change the outcome of the operations [i.e.,  $(a + b) + c = a + (b + c)$  and  $(ab)c = a(bc)$ ].

**Box-and-Whisker Plot:** A graphic method for showing a summary of data using median, quartiles and extremes of data.



**Central Tendency:** The degree of clustering of the values of a statistical distribution that is usually measured by the arithmetic mean, mode, or median.

**Combination:** A subset of the elements in a given set, without regard to the order in which those elements are arranged.

**Commutative Property:** A property of addition or multiplication in which the sum or product stays the same when the order of the addends or factors is changed (i.e.,  $a + b = b + a$  and  $ab = ba$ ).

**Complementary Event:** The opposite of an event. That is, the set of all outcomes of an experiment that are not included in an event.  $P(A^{\wedge}) = 1 - P(A)$ .

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>Complex Numbers:</b>	Any number, real or imaginary, of the form $a + bi$ , where $a$ and $b$ are real numbers and $i = -1$ .
<b>Composite Number:</b>	Any positive integer exactly divisible by one or more positive integers other than itself and 1.
<b>Compound Event:</b>	An event that consists of two or more simple events; for example: A or B; A and B and C.
<b>Congruent:</b>	Having the same shape and the same size.
<b>Conjecture:</b>	A statement believed to be true but not proved.
<b>Contrapositive:</b>	The implication which results from replacing the antecedent by the negation of the consequent and the consequent by the negation of the antecedent.
<b>Converse:</b>	The theorem (or implication) resulting from interchanging the hypothesis and conclusion.
<b>Coordinate System:</b>	A method of locating points in the plane or in space by means of numbers. A point in the plane is located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.
<b>Correlation :</b>	A measure of the mutual relationship between two variables.
<b>Customary System:</b>	A system of weights and measures frequently used in the United States. The basic unit of weight is the pound; the basic unit of capacity is the quart.
<b>Deductive Reasoning:</b>	The process of reasoning from statements accepted as true to reach a conclusion.
<b>Dependent Event:</b>	The probability of one event depends on the outcome of another event.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>Direct Proof:</b>	Uses an argument that makes direct use of the hypothesis and arrives at the conclusion.
<b>Direct Variation:</b>	Two variables are so related that their ratio remains constant.
<b>Distributive Property:</b>	Multiplication is distributive over addition and subtraction [i.e., $a(b + c) = ab + ac$ and $a(b-c) = ab-ac$ ].
<b>Domain:</b>	The set of all possible values for the unknown in an open sentence.
<b>Equation:</b>	A statement of equality between two mathematical expressions (e.g., $X + 5 = Y - 2$ ).
<b>Equivalent Forms:</b>	Different forms of numbers that name the same number (e.g., fraction, decimal, percent as $\frac{1}{2}$ , .5, 50%).
<b>Expanded Notation:</b>	Involves writing the number in expanded form to show the value of each digit (e.g., $15,629 = 10,000 + 5,000 + 600 + 20 + 9$ ).
<b>Exponent:</b>	A numeral used to tell how many times a number or variable is used as a factor (e.g., $a^2$ , $2^n$ , $y^x$ ).
<b>Exponential Function:</b>	A function whose general equation is $y = a \times b^x$ or $y = a \times b^{kx}$ , where a, b and k stand for constants.
<b>Exponential Notation:</b>	A way of writing numbers using bases and exponents [e.g., $425 = (4 \times 10^2) + (2 \times 10^1) + (5 \times 10^0)$ ].
<b>Expression:</b>	A mathematical phrase that can include operations, numerals and variables. In algebraic terms: $2l + 3x$ ; in numeric terms: $13.4 - 4.7$ .
<b>Factor:</b>	The number or variable multiplied in a multiplication expression.
<b>Factorial:</b>	The expression $n!$ (n factorial) is the product of all the numbers from 1 to n for any positive integer n.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>Function:</b>	A relation in which each value of an independent variable is associated with a unique value of the dependent value.
<b>Geoboard:</b>	A board with pegs aligned in grid fashion that permits rubber bands to be wrapped around pegs to form geometric figures.
<b>Graphing Calculator:</b>	A calculator that will store and draw the graphs of several functions at once.
<b>Histogram:</b>	A graphical display that shows frequencies as bars. It shows what proportion of cases fall into each of several categories.
<b>Identity Property:</b>	In addition, adding 0 to a number leaves the number unchanged, or identical ( $2+0=2$ ). Hence, zero is the identity element for addition. In multiplication, the product of any number and 1 is that number ( $4 \times 1=4$ ). Hence, one is the identity element for multiplication.
<b>Independent Events:</b>	Events such that the outcome of the first event has no effect on the probabilities of the outcome of the second event (e.g., two tosses of the same coin are independent events).
<b>Inductive Reasoning:</b>	Generalizations made from particular observations in a common occurrence.
<b>Inequality:</b>	A mathematical sentence that contains a symbol (e.g., $>$ , $<$ , $\geq$ , $\leq$ or $\neq$ ) in which the terms on either side of the symbol are unequal (e.g., $x < y$ , $7 > 3$ , $n \geq 4$ ).
<b>Infinite:</b>	Has no end or goes on forever.
<b>Integer:</b>	A number that is a positive whole number, a negative whole number or zero.
<b>Interquartile Range:</b>	The inter-quartile range is a measure of the spread of or dispersion within a data set. It is calculated by taking the difference between the upper and the lower quartiles.
<b>Inverse:</b>	A new conditional formed by negating both the antecedent and the consequent of a conditional.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>Inverse Function:</b>	Formally, suppose $f$ is a one-to-one function with a domain $A$ and range $B$ . The inverse function $f^{-1}$ is a function with these properties: <ul style="list-style-type: none"><li>▪ <math>f^{-1}</math> has domain <math>B</math> and range <math>A</math>.</li><li>▪ For all <math>x</math> in <math>B</math>, <math>f(f^{-1}(x)) = x</math>, or <math>f \circ f^{-1} = x</math></li></ul> Informally, if $f$ is a function from $A$ to $B$ , then an inverse for $f$ is a function from $B$ to $A$ , with the property that a round trip (a composition) from $A$ to $B$ to $A$ returns each element to itself.
<b>Inverse Operations:</b>	Operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations).
<b>Inverse Variation:</b>	When the ratio of one variable to the reciprocal of the other is constant, one of them is said to vary inversely as the other.
<b>Inverse Statement:</b>	Formed by negating the hypothesis and negating the conclusion of the original statement. <i>If <math>p</math>, then <math>q</math></i> becomes <i>if not <math>p</math>, then not <math>q</math></i> .
<b>Irrational Number:</b>	A number that cannot be written as a simple fraction. It is an infinite and non-repeating decimal.
<b>Limit:</b>	A number to which the terms of a sequence get closer so that beyond a certain term all terms are as close as desired to that number.
<b>Line of Best Fit:</b>	The line that fits a set of data points with the smallest value for the sum of the squares of the errors (vertical distances) from the data points to the line; the regression line.
<b>Linear Function:</b>	A function whose general equation is $y = mx + b$ , where $m$ and $b$ stand for constants and $m \neq 0$ .
<b>Linear Measurement:</b>	Measurement in a straight line.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>Logarithm:</b>	The exponent indicating the power to which a fixed number, the base, must be raised to produce a given number. For example, if $n^x = a$ , the logarithm of a, with n as the base, is x; symbolically, $\log_n a = x$ . If the base is 10, the log of 100 is 2.
<b>Logarithmic Function:</b>	Any function in which an independent variable appears as the argument in the form of a logarithm; they are the inverse functions of exponential functions.
<b>Manipulatives:</b>	Materials that allow students to explore mathematical concepts in a concrete mode.
<b>Mathematical Model:</b>	A representation in the mathematical world of some phenomenon in the real world. It frequently consists of a function or relation specifying how two variables are related.
<b>Matrix:</b>	A rectangular array of numbers representing such things as the coefficients in a system of equations arranged in rows and columns.
<b>Maximum:</b>	The greatest number in a set of data.
<b>Mean:</b>	The sum of the set of numbers divided by n, the number of numbers in the set.
<b>Median:</b>	The number that lies in the middle when a set of numbers is arranged in order. If there are two middle values, the median is the mean of these values.
<b>Metric System:</b>	A system of measurement used throughout the world based on factors of 10. It includes measures of length, weight and capacity.
<b>Minimum:</b>	The least number in a set of data.
<b>Missing Addend:</b>	A member of an addition number sentence in which that term is missing (e.g., $5 + \underline{\quad} = 8$ ).
<b>Mode:</b>	The number(s) that occurs most often in a set of numbers (e.g., in the set 1, 2, 3, 3, 5, 8; the mode is 3).
<b>Models:</b>	Models are representations of real world phenomena. Numerical tables, graphs, equations or systems of equations are all mathematical representations that may be used



# ELEMENTARY STANDARDS

## Grades Pre-K-3

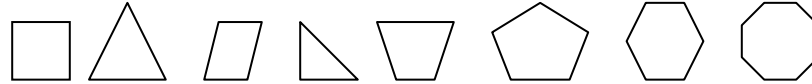
### Mathematics

to model real world phenomena. A good model should capture the essential character of whatever is being modeled.

- Multiple:** A number that is the product of a given integer and another integer (e.g., 6 and 9 are multiples of 3).
- Normal Curve:** A graphical plot of a mathematical function (frequency distribution) which is unimodal and symmetrical.
- One-to-one Correspondence:** When one and only one element of a second set is assigned to an element of a first set, all elements of the second set are assigned, and every element of the first set has an assignment, the mapping is called one-to-one (e.g., in the set Bill Clinton, George Bush, Ronald Reagan, Jimmy Carter, Hillary Clinton, Barbara Bush, Nancy Reagan and Rosalynn Carter, there is a one-to-one correspondence between the pairs).
- Open Sentence:** A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown (e.g.,  $x + 5 = 9$ ,  $y - 2 = 7$ ).
- Order of Operations:** Rules for evaluating an expression: work first within parentheses; then calculate all powers, from left to right; then do multiplications or divisions, from left to right; then do additions and subtractions, from left to right.
- Patterns:** Regularities in situations such as those in nature, events, shapes, designs and sets of numbers (e.g., spirals on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12,...).
- Permutation:** An arrangement of a given number of objects from a given set in which the order of the objects is significant.
- Perpendicular Lines:** Two lines that intersect to form right angles (e.g.,  $\perp$ ,  $\lrcorner$ ,  $\ulcorner$ ).
- Plotting Points:** Locating points by means of coordinates, or a curve by plotted points, representing an equation by means of a curve so constructed.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

**Polygon:** A union of segments connected end to end, such that each segment intersects exactly two others at its endpoints.



**Powers:** A number expressed using an exponent. The number  $5^3$  is read five to the third power or five cubed.

**Prime:** An integer greater than one whose only positive factors are 1 and itself (e.g., 2, 3, 5, 7, 11, 13, 17, and 19).

**Probability:** A number from 0 to 1 that indicates how likely something is to happen.

**Problem-Solving:** Finding ways to reach a goal when no routine path is apparent.

**Proof by Contradiction:** A proof in which, if  $s$  is to be proven, one reasons from not- $s$  until a contradiction is deduced; from this it is concluded that not- $s$  is false, which means that  $s$  is true.

**Proportion:** An equation of the form  $\frac{a}{b} = \frac{c}{d}$  that states that the two ratios are equivalent.

**Quadrilateral:** A four-sided polygon.

**Quartiles:** The three values that divide an ordered set into four subsets of approximately equal size. The second quartile is the median.

**Radian:** A unit of angular measure equal to  $\frac{1}{2\pi}$  of a complete revolution.

**Range (1):** The difference between the greatest number and the least number in a set of data.

**Range (2):** The set of output values for a function.

**Rate of Change:** The limit of the ratio of an increment of the function value at the point to that of the independent variable as the increment of the variable approaches zero.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

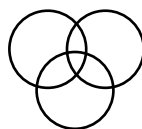
<b>Ratio:</b>	A comparison of two numbers by division.
<b>Rational numbers:</b>	Any number that can be written in the form $\frac{a}{b}$ where $a$ is any integer and $b$ is any integer except zero.
<b>Real Numbers:</b>	The set consisting of all rational numbers and all irrational numbers.
<b>Reasonableness:</b>	Quality of a solution such that it is not extreme or excessive.
<b>Reciprocal:</b>	The fractional number that results from dividing one by the number.
<b>Rectangular Prism:</b>	A three-dimensional figure whose sides are all rectangles; a box.
<b>Reflection:</b>	A transformation that produces the mirror image of a geometric figure.
<b>Regression:</b>	The line that represents the least deviation from the points in a scatter plot of data.
<b>Regular Polygon:</b>	A polygon in which all sides have the same measure and all angles have the same measure.
<b>Relation:</b>	A set of ordered pairs.
<b>Reliability:</b>	The extent to which a measuring procedure yields the same results on repeated trials.
<b>Repeated Addition:</b>	A model for multiplication (e.g., $2 + 2 + 2 = 3 \times 2$ ).
<b>Rotation:</b>	A transformation that maps every point in the plane to its image by rotating the plane around a fixed point or line.
<b>Scatterplot:</b>	A graph of plotted points that show the relationship between two sets of data.
<b>Scientific Calculator:</b>	A calculator that represents very large or very small numbers in <b>scientific notation</b> with the powering, factorial, square root, negative and reciprocal keys.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

<b>Scientific Notation:</b>	A way in writing a number of terms of an integer power of 10 multiplied by a number greater than or equal to 1 and less than 10.
<b>Sequence:</b>	A set of ordered quantities (e.g., positive integers).
<b>Series:</b>	The indicated sum of the terms of a sequence.
<b>Similarity:</b>	Having the same shape but not necessarily the same size.
<b>Simple Event:</b>	An event whose probability can be obtained from consideration of a single occurrence (e.g., the tossing of a coin is a simple event).
<b>Simulation:</b>	Modeling a real event without actually observing the event.
<b>Slope:</b>	The slope of a line is the ratio of the change in y to the corresponding change in x; the constant m in the linear function equation; rise/run.
<b>Standard Deviation:</b>	The square root of the variance.
<b>Stem-and-Leaf Plot:</b>	A frequency distribution made by arranging data (e.g., student scores on a test were 98, 96, 85, 93, 83, 87, 85, 87, 93, 75, 77 and 83). This data is displayed in a stem-and-leaf plot below.  $\begin{array}{r l} 9 & 8, 6, 3, 3 \\ 8 & 7, 7, 5, 5, 3, 3 \\ 7 & 7, 5 \end{array}$
<b>Systems of Equations:</b>	Two or more equations that are conditions imposed simultaneously on all the variables, but may or may not have common solutions (e.g., $x + y = 2$ , and $3x + 2y = 5$ ).
<b>Symmetry:</b>	A line of symmetry separates a figure into two congruent halves, each of which is a reflection of the other (e.g., $\emptyset$ , the line through the center of the circle divides it into congruent halves).
<b>T-Test:</b>	A statistical test done to test the difference of means of two samples.

ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

- Tessellation:** A repetitive pattern of polygons that covers an area with no holes and no overlaps (e.g., floor tiles).
- Transformation:** An operation on a geometric figure by which each point gives rise to a unique image.
- Translation:** A transformation that moves a geometric figure by sliding each of the points the same distance in the same direction.
- Tree Diagram:** A diagram used to show the total number of possible outcomes in a probability experiment.
- Trigonometric Functions:** A function (e.g., sine, cosine, tangent, cotangent, secant, cosecant) whose independent variable is an angle measure, usually in degrees or radians.
- Valid Argument:** An argument with the property that no matter what statements are substituted in the premises, the truth value of the form is true. If the premises are true, then the conclusion is true.
- Variable:** A symbol used to stand for any one of a given set of numbers or other objects (e.g., in the equation  $y = x + 5$ ,  $y$  and  $x$  are variables).
- Variance:** In a data set, the sum of the squared deviations divided by one less than the number of elements in the set (sample variance  $s^2$ ) or by the number of elements in the set (population variance  $\sigma^2$ ).
- Vector:** A quantity that has both magnitude and direction (e.g., physical quantities such as velocity and force).
- Venn Diagram:** A display that pictures unions and intersections of sets.



ELEMENTARY STANDARDS  
Grades Pre-K-3  
Mathematics

- Volume:** The amount of space enclosed in a space (3-dimensional) figure, measured in cubic units.
- Y-Intercept:** The y-intercept of a line is the y-coordinate of the point at which the graph of an equation crosses the y-axis.
- $\pi$ :** pi, the ratio of the circumference of a circle to its diameter: 3.1415926535.